

# Development of Areal Wavelet Transform applying to the 2D image

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Today, there is an increased need for quality control in the manufacturing sectors. Particularly, the automated detection of the product surface defect is essential to fulfill the requirement of quality assurance.

In this research, the algorithm of surface defect detection by use of wavelet transform has been developed. This method is established by one-dimensional wavelet filter. However, the image processing of 2D image for defect detection should be applied by two-dimensional filter. Therefore, in this report, a new wavelet transform is developed with 2D filter and the method is called for areal wavelet transform. This areal wavelet transform is proposed and is confirmed the efficiency of denoising 2D image.

**Key words:** wavelet transform, Haar wavelet, areal wavelet transform

## 1. Introduction

Most recently, wavelet analysis stands out to be a powerful tool for signal processing including data compression, data transmission and denoising. Similar to moving average, mathematical calculation of wavelet transform (WT) falls in a form of convolution. In the present wavelet image processing, two-dimensional filter processing is enabled by processing a one-dimensional filter to a horizontal direction and perpendicular both to a two-dimensional object image. However, it is rational that they are two dimensions as for the filter which processes a two-dimensional object image. Therefore, in this research, the two-dimensional filter is designed and proposed and it is called areal wavelet transform.

## 2. Conventional wavelet transform

In this research, wavelet transform has been investigated for denoising and defect detection. For denoising and defect detection, the wavelet transformation is a powerful tool because it employs time-frequency windows to decompose the original signal into a sequence of subsets in scalar space as shown in Fig. 1<sup>1)</sup>. As the wavelet functions are compactly supported in both spatial and frequency domains, the different components in the scalar space can be decomposed and reconstructed perfectly. The nature of wavelet transformation also reserves both frequency and spatial properties. Wavelet transform is thus able to give not only the amplitude of a specific spatial frequency (vertical level -1, -2, -3... in Fig. 1), but also the location of corresponding spatial frequency (horizontal axis in Fig. 1).

In this paper, conventional Daubechies wavelet is used as the scaling and wavelet functions as following equations (1), (2).

$$\psi(t) = \begin{cases} \sqrt{2}/2 & 0 \leq t < 1/2 \\ -\sqrt{2}/2 & 1/2 \leq t < 1 \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

$$\phi(t) = \begin{cases} \sqrt{2}/2 & 0 \leq t < 1 \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

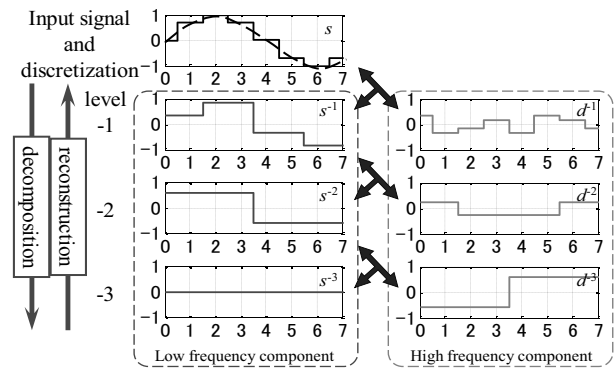


Fig. 1 Concept of wavelet transform

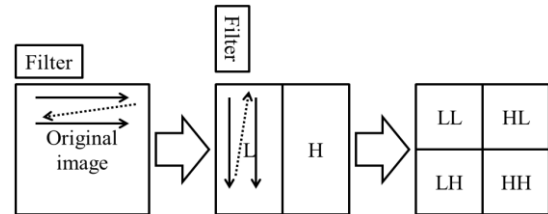


Fig. 2 Process of conventional 2D-Haar wavelet transform

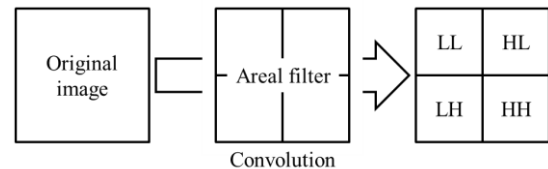


Fig. 3 Process of areal Haar wavelet transform

As shown in Fig. 2, in conventional wavelet transform algorithm for 2D image, the decomposition and reconstruction process are applied in row direction first and in column direction by turns. Then, the results components are four component (LL, HL, LH, HH). Next level components is obtained by processing LL component similarly. Thus, this algorithm is computationally expensive.

### 3. Areal wavelet transform

In this research, the areal wavelet transform is introduced for 2D image wavelet transform. In Fig. 3, the concept of areal wavelet transform is shown. In this figure, 2D areal filter is formed by the tensor product of two one-dimensional mother wavelets in orthogonal directions. In comparison to conventional wavelet transform, this algorithm is only performed by convolution between input original image and 2D areal filter. In this paper, 2D areal filter is formed by the tensor product of Daubechies mother wavelet.

In order to confirm the validity of this 2D filters, difference between the input image and reconstructed image is investigated. Figure 4 shows the original image and the image with additional noise. In Fig. 5, the histogram of difference between original image and reconstructed image is shown. From this figure, it is found that the differences are almost zero. Therefore, the proposed 2D filters are valid for wavelet transform filter.

### 4. Denoising results for 2D image

In this section, the denoising capability of areal wavelet transform is confirmed. The areal wavelet transform algorithm is compared with conventional wavelet transform algorithm. As the input signal is decomposed into scalar domain, WT further provides the capability of removing such a component that has amplitude less (or larger) than a specified threshold. For the purpose of reducing small-amplitude noises, the filtering algorithm performs decomposition and reconstruction by setting the absolute value of scalar coefficients smaller than the unwanted amplitude to zero. In Fig. 6, the algorithm of denoising 2D image by wavelet transform is shown. In this paper, the HH component is only filtered by threshold value  $A_{L1} \sim A_{L2}$  in Fig. 6. In Fig. 7, the results of denoising by conventional wavelet transform and areal wavelet transform are shown. In comparison to input image shown in Fig. ??, the denoising result by conventional wavelet transform is reduced noise. However, whole color is faded out. Therefore, this result is filtered not only noise but also original image. On the other hand, the denoising result by areal wavelet transform is also reduced noise, and whole color of this result is near the original image. As a result, the proposed areal wavelet transform is better than conventional wavelet transform.

### 5. Conclusion

In this paper, in order to introduce the 2D filter for denoising 2D image the areal wavelet transform is proposed. The proposed areal wavelet transform filter is able to decompose and reconstruct the image completely. Compared to the filters with conventional wavelet transform, the new 2D filter can separate the noise from 2D image much more efficiently.

#### References

- 1) Jiang, X., 2004, Third generation of morphological features from micro and nano scalar surfaces, pp.1235-1240.

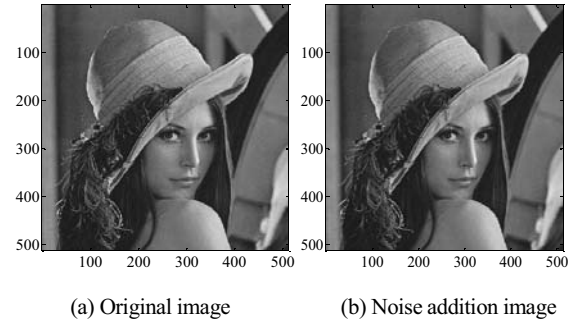


Fig. 4 Original image and noise addition image

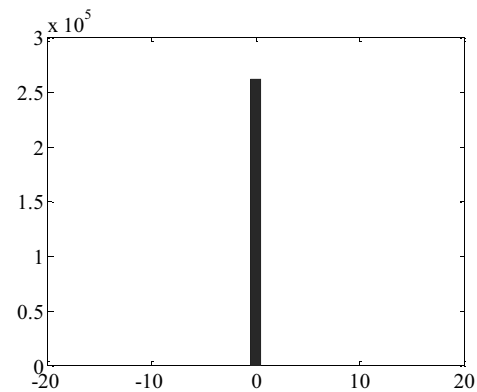


Fig. 5 Difference between original image and reconstructed image

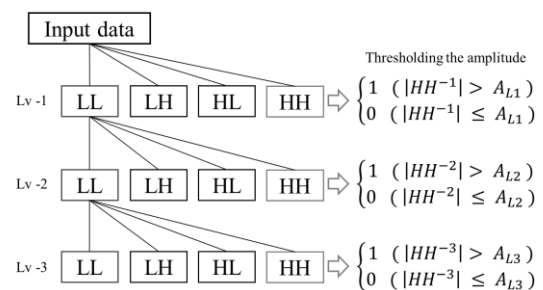


Fig. 6 A method of denoising

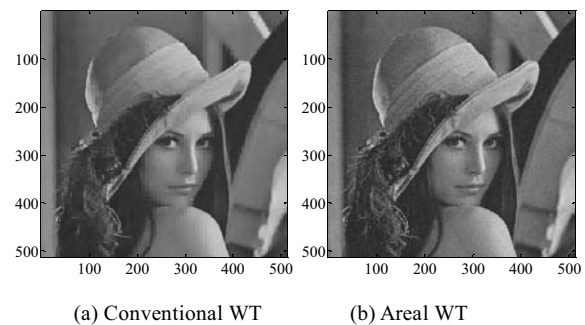


Fig. 7 Denoising results for including noise image with conventional Haar wavelet transform and areal Haar wavelet transform